

Yukawa Matrix for the Neutrino and Lepton Flavour Violation *

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We estimate the magnitude of Lepton Flavour Violation (LFV) from the phase of the neutrino Yukawa matrix. In the minimal supersymmetric standard model with right-handed neutrinos, the LFV processes $l_i \rightarrow l_j \gamma$ can appear through the slepton mixing, which comes from the renormalization group effect on the right-handed neutrino Yukawa interaction between the Grand Unified Theory scale and the heavy right-handed neutrino mass scale. Two types of phases exist in the neutrino Yukawa matrix. One is the Majorana phase, which can change the magnitude of the LFV branching ratios by a few factor. The other phases relate for the size of the Yukawa hierarchy and its phase effect can change the LFV branching ratios by several orders of magnitude.

1. Introduction

Lepton flavour violation (LFV) is an important signature into physics beyond the standard model (SM). In the SM with massive neutrinos, neutrinos have the Yukawa interaction. Lepton flavour is violated like quark flavour. However, LFV is strongly suppressed by the neutrino masses. Supersymmetry (SUSY) with massive neutrinos makes the situation drastically changed. It can predict sizable LFV effects, because the alternative source of LFV is generated from the slepton mixing through the renormalization group effect on the right-handed neutrino Yukawa interaction.

The LFV processes $l_i \rightarrow l_j \gamma (i \neq j)$ of charged lepton are being measured. The MEG experiment [1] gives the upper bound on the $\mu \rightarrow e \gamma$ process as

$$\text{Br}(\mu \rightarrow e \gamma) \leq 1.2 \times 10^{-11}. \quad (1)$$

The forthcoming experiment reaches to $\mathcal{O}(10^{-14})$. The τ decay processes are also measured at B-factories [2]. On the other hand, it has been already discovered that lepton flavour is violated in the neutrino sector. The oscillation parameters will be measured in detail.

In this letter, we study the phase of the neutrino Yukawa matrix and investigate its effect on the magnitude of the LFV processes.

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2. The neutrino Yukawa matrix

In the framework based on the see-saw mechanism, we can parameterize the neutrino Yukawa matrix Y_ν in terms of physical quantities as [3]

$$\frac{v_u}{\sqrt{2}} Y_\nu = \sqrt{M_R^{diag}} \mathcal{R} \sqrt{m_\nu^{diag}} U_{MNS}^\dagger, \quad (2)$$

where v_u is the vacuum expectation value of the Higgs boson, U_{MNS} is the observed Maki-Nakagawa-Sakata (MNS) matrix including two Majorana phases $\xi_{1,2}$; i.e.,

$$U_{MNS} \sim \begin{pmatrix} 0.85 & -0.53 & 0 \\ 0.37 & 0.60 & -0.71 \\ 0.37 & 0.60 & 0.71 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\xi_1} & \\ & & e^{i\xi_2} \end{pmatrix}. \quad (3)$$

Here we neglect the $(1-3)$ element of U_{MNS} . m_ν^{diag} is the neutrino mass matrix and M_R^{diag} is the right-handed Majorana neutrino mass matrix in each diagonal base. An arbitrary complex orthogonal matrix \mathcal{R} is expressed as

$$\mathcal{R} \equiv O_{12} O_{23} O_{31} Q_{12} Q_{23} Q_{31}, \quad (4)$$

where

$$O_{12}(\theta_{12}) \equiv \begin{pmatrix} \cos \theta_{12} & -\sin \theta_{12} & \\ \sin \theta_{12} & \cos \theta_{12} & \\ & & 1 \end{pmatrix}, \text{etc.}, \quad (5)$$

and

$$Q_{ij} \equiv O_{ij}(i\eta_{ij}). \quad (6)$$

The elements Q_{ij} give a drastic effect on structure of the Yukawa matrix as hyperbolic functions. We call $i\eta_{ij}$ as *R-phase* to distinguish it from the Majorana phase.

In the following, we assume that the neutrino masses are *approximately* degenerate and the right-handed neutrino masses are degenerate

$$m_\nu^{diag} \sim m \begin{pmatrix} 1 & & \\ & 1 + \frac{\Delta m_\odot^2}{2m^2} & \\ & & 1 + \frac{\Delta m_\oplus^2}{2m^2} \end{pmatrix}, \quad (7)$$

$$M_R^{diag} \sim M_R \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}, \quad (8)$$

where, Δm_\odot^2 and Δm_\oplus^2 are the solar and atmospheric neutrino mass squared differences, M_R is the size of right-handed neutrinos, and m is undetermined light neutrino mass parameter. We then obtain the neutrino Yukawa matrix Y_ν in its diagonal base as

$$Y_\nu^{diag} \sim \frac{\sqrt{2}}{v_u} \sqrt{M_R m} \begin{pmatrix} 1/r & & \\ & 1 & \\ & & r \end{pmatrix}, \quad (9)$$

where

$$r^2 \equiv 2x^2 - 1 + 2x\sqrt{x^2 - 1}, \quad (10)$$

$$x \equiv \cosh \eta_{12} \cosh \eta_{23} \cosh \eta_{31}. \quad (11)$$

This expression indicates the characteristic relation ($y_1/y_2 = y_2/y_3$) among the neutrino Yukawa couplings, which is similar to those of the quarks and the charged leptons ($m_u/m_c \sim m_c/m_t$, etc.). For $m \sim 0.1\text{eV}$ and $M_R \sim 10^9\text{GeV}$, r is close to $\mathcal{O}(10^2)$, so that the neutrino Yukawa couplings become hierarchical. The parameter r is written in terms of the combination of *R-phase* (see Eqs.(10) and (11)), and it determines the size of the neutrino Yukawa hierarchy.

3. Lepton Flavour Violation

We consider the SUSY models, especially the minimal supersymmetric standard model

(MSSM) with right-handed neutrinos. In the slepton sector, the MSSM Lagrangian has an alternative source of LFV through the following soft SUSY breaking terms,

$$-\mathcal{L}_{soft} = \left(A_{ij}^e H_d \tilde{e}_{Ri}^* \tilde{L}_j + A_{ij}^\nu H_u \tilde{\nu}_{Ri}^* \tilde{L}_j + \text{h.c.} \right) + (m_{\tilde{L}}^2)_{ij} \tilde{L}_i^\dagger \tilde{L}_j + (m_{\tilde{e}}^2)_{ij} \tilde{e}_i^* \tilde{e}_j + (m_{\tilde{\nu}}^2)_{ij} \tilde{\nu}_{Ri}^* \tilde{\nu}_{Rj}, \quad (12)$$

where $A^{e,\nu}$ are the slepton tri-linear couplings and $m_{\tilde{L},\tilde{e},\tilde{\nu}}$ are the soft mass parameters for the sleptons. We assume that these couplings are universal at the Grand Unified Theory (GUT) scale (M_{GUT}), i.e.,

$$(m_{\tilde{L}}^2)_{ij} = (m_{\tilde{e}}^2)_{ij} = (m_{\tilde{\nu}}^2)_{ij} = \delta_{ij} m_0^2 \\ A^\nu = Y_\nu a_0 m_0, \quad A^e = Y_e a_0 m_0. \quad (13)$$

In this framework, the LFV processes $l_i \rightarrow l_j \gamma (i \neq j)$ can appear due to the slepton mixing [4], which comes from the renormalization group effect on Y_ν between the scales of M_{GUT} and M_R

$$(\Delta m_{\tilde{L}}^2)_{ij} \sim -\frac{1}{16\pi} (6 + 2a_0) m_0^2 (Y_\nu^\dagger Y_\nu)_{ij} \log \frac{M_{GUT}}{M_R}. \quad (14)$$

The branching ratios for these processes are expressed by

$$\text{Br}(l_i \rightarrow l_j \gamma) \sim \frac{\alpha^3}{G_F^2} \frac{|(\Delta m_{\tilde{L}}^2)_{ij}|}{m_S^8} \tan^2 \beta, \quad (15)$$

where m_S is the typical SUSY scale, $\tan \beta$ is the ratio of the vacuum expectation values of Higgs bosons, α is the fine structure constant, and G_F is Fermi constant.

Let us consider the r dependence of LFV with $\eta_{23,31} = 0$, the effect of *R-phase* contributes to only $\mu \rightarrow e \gamma$ process. A comparison of each magnitude of LFV processes is already meaningless, and we plotted the branching ratio for $\mu \rightarrow e \gamma$ process (see Figure 1.), where we take $m_S = m_0 = 1\text{TeV}$, $a_0 = 1$, $\tan \beta = 10$, and $M_{GUT} = 2 \times 10^{16}\text{GeV}$. The LFV branching ratio is approximately proportional to r^4 , so that a large value of r can change the branching ratio by several orders of magnitude.

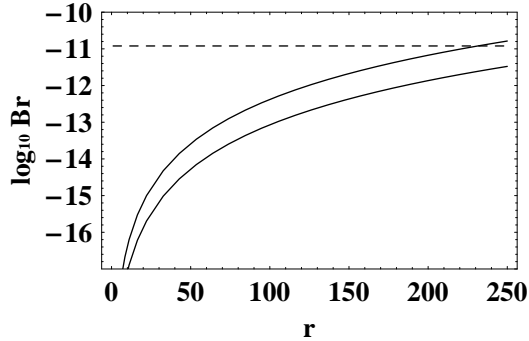


Figure 1. The branching ratio of the $\mu \rightarrow e\gamma$ event for $\alpha = \pi$ (upper solid line) and $\alpha = \pi/2$ (lower solid line). The experimental upper limit from MEG is also shown (dashed line).

On the other hand, we show the dependence on the Majorana phase ξ_1 (see Figure 2.). When $\eta_{23,31} = 0$, the branching ratio does not depend on ξ_2 . Figure 2 shows that the branching ratio is a periodic function on ξ_1 , and that the effect of Majorana phase is smaller than that of r .

4. Summary

We have analyzed the structure of the neutrino Yukawa matrix and have discussed the magnitude of LFV processes from the phase effect on the neutrino Yukawa matrix in the MSSM with the right-handed neutrinos.

The neutrino Yukawa matrix has two types of phases, Majorana phases and R -phases. In the case that neutrino masses are degenerate and the right-handed neutrino masses are degenerate, the eigenvalues of the neutrino Yukawa matrix become hierarchical spectrum and the R -phases determine the size of Yukawa hierarchy.

In the SUSY models, sizable LFV can arise due to the slepton mixing from the renormalization group effect on the neutrino Yukawa matrix between M_{GUT} and M_R . The Majorana phases can change the LFV branching ratios by a factor, and these magnitudes become periodic as the function of Majorana phases. On the other hand, R -phases enhance the magnitude of the LFV branching ra-

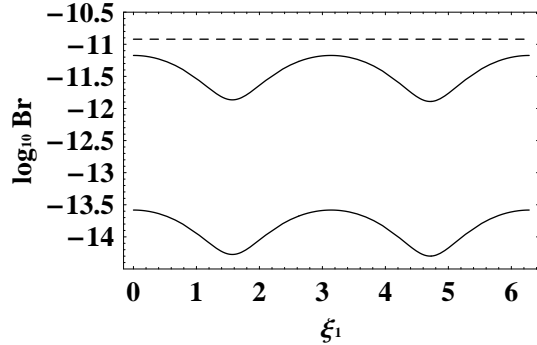


Figure 2. The branching ratio of the $\mu \rightarrow e\gamma$ event for $r = 200$ (upper solid line) and $r = 50$ (lower solid line). The upper limit is also shown (dashed line).

tios by several orders.

REFERENCES

1. MEG Web site, <http://meg.psi.ch/>.
2. K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **92**, 171802 (2004) [arXiv:hep-ex/0310029].
3. S. Pascoli, S. T. Petcov and C. E. Yaguna, Phys. Lett. B **564** 241 (2003).
4. J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D **53**, 2442 (1996)
J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B **357**, 579 (1995)
5. S. Kanemura, K. Matsuda, T. Ota, T. Shindou, E. Takasugi and K. Tsumura, arXiv:hep-ph/0409283.